

TEACHING STUDENTS INTERNET SAFETY THROUGH AN ARTIFICIAL INTELLIGENCE MOBILE APPLICATION



Co-funded by the
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of the European Union

ADDITIONAL MATERIALS FOR TEACHING MATHEMATICS



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ENTRY

Serious Games in Teaching Mathematics - Learning by Playing!

Learning math can sometimes be challenging for students who often feel aversion to the subject. However, thanks to advancing educational technologies, there is a new and exciting tool that has the potential to transform the way students learn math - serious games. Serious games are interactive computer applications that combine game elements with learning, allowing students to learn while having fun. In today's article, we will look at the benefits of using serious games in teaching mathematics.

1. **An engaging and motivating form of learning:** Serious games engage students by introducing game elements into the learning process. As a result, students are more likely to engage in math learning as they have the chance to explore, discover and achieve in an interactive and engaging way. Game elements such as competition, prizes, difficulty levels and discovering new worlds stimulate the motivation of students to self-improve and acquire mathematical knowledge.
2. **Improve logical thinking skills:** Serious math teaching games develop students' logical thinking skills. Math games often require analysis, problem solving, decision making, and critical thinking. Students must use math strategies and techniques to achieve the goals of the game. In this way, they develop their logical reasoning skills, which is applicable not only in mathematics, but also in other areas of life.
3. **Individualization of the learning process:** Serious games allow you to adjust the level of difficulty to the individual needs and skills of students. Math games often offer different levels of difficulty, making it possible to match the challenges to the abilities of each student. Students can learn at their own pace, develop skills at the appropriate level, and advance to the next level of difficulty as they progress.
4. **Practical Mathematics:** Serious games allow students to practically apply math in the context of real-life situations. Math games often present problems

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and challenges that require the use of mathematics to solve them. Students can experiment, explore, create strategies and test their math skills in practice. This enables students to understand how math applies to everyday life and motivates them to learn.

5. Collaboration and Interaction: Serious games often offer opportunities for collaboration and interaction between students. Students can solve math problems, make decisions and solve problems together. This collaboration develops communication and teamwork skills and teaches students how to collaborate and exchange ideas effectively.

Summary: Serious games are an innovative math teaching tool that transforms the traditional learning process into an interactive and engaging adventure. Through the elements of games, students have the opportunity to discover, experiment and acquire mathematical knowledge in an attractive and motivating way. Serious games develop logical thinking skills, enable the individualization of the learning process, combine mathematics with practical application and promote cooperation between students. That is why it is worth using serious games in teaching maths, so that students can learn through play and develop their math skills in an innovative and interactive way.

Mathematics is an extremely important subject that develops logical thinking, problem solving and data analysis skills. However, students may often find it difficult to understand abstract mathematical concepts.

iSafetyApp is an interactive web application that integrates various elements of mathematics into a secure online environment. The app can be used as an introduction to a new math topic.

It is worth starting or ending a lesson using an application that will allow students to take a trip to the virtual world in which they really like to be.

Instruct students to download the app:

Safety Escape Game - Apps on Google Play

<https://play.google.com/store/apps/details?id=com.bgs.safetyescapegame>

Developed as part of the Erasmus + project, the guide contains additional materials for teaching selected mathematical topics. We encourage you to using sets of tasks, teaching tips, suggestions for using mathematical issues to solve various problems.

MATHEMATICS DIFFERENTLY

ALGEBRA



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1. Applying algebra to everyday life: Give students different examples of real-life situations where algebra can be applied. For example, ask them to solve a household budget problem, calculate interest, or interpret statistical data. Through this, students will see the practical application of algebra and understand how it can help in their daily lives.
2. Equation Board Game: Create an interactive board game where students have to solve equations to move around the board. Each field on the board can contain an equation, and students must solve it to move on to the next field. This method of teaching algebra allows students to practice solving equations in a fun and engaging way.
3. Research Project: Ask students to do a research project that involves the use of algebra. For example, they can study plant growth depending on the amount

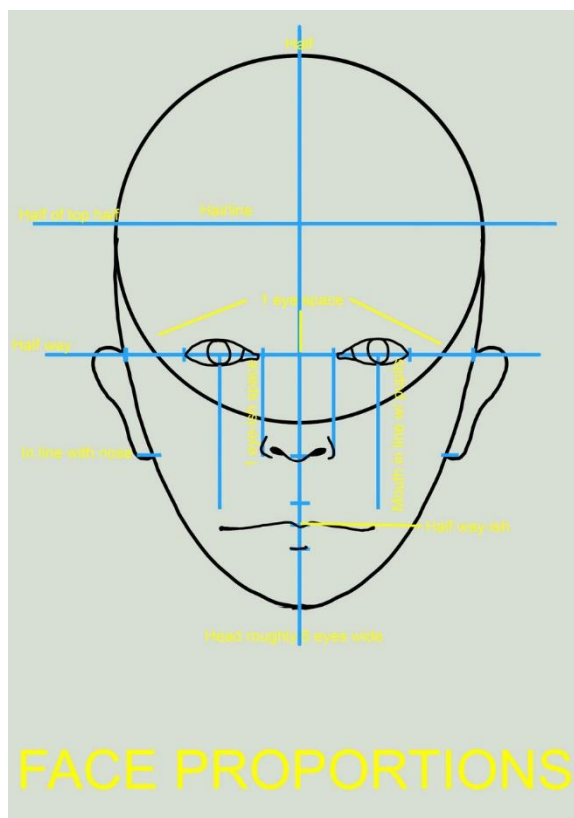
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of sunlight, temperature, and amount of water. Students will need to collect data and analyze it using algebra to find the right patterns and relationships.

4. Computer Simulations: Use computer simulation software that allows students to experiment and see how changes in algebraic equations affect the results. For example, they can simulate the motions of the planets in the solar system and experiment with various parameters such as planetary masses and distances. This interactive approach will allow students to see in practice how algebra can be applied to model and predict various phenomena.

5. Real-life problem tasks: Prepare a set of problem tasks that relate to real-life situations such as financial problems, spatial geometry, travel itineraries, etc. Students will have to identify the relevant equations and solve them to find a solution to the problem. This hands-on approach to learning algebra will enable students to see how it can be applied to a variety of life contexts.



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PROPORTIONS

1. Culinary proportion design: Ask students to prepare a recipe for a dish, but with restrictions on the proportions of the ingredients. For example, they may need to make muffins, but the proportions of flour, sugar, and butter will need to be maintained. This will help students understand the importance of proportions and how they affect the final product.

2. Studying proportions in architecture: Suggest a project for students to study proportions in buildings and architectural structures. They can study the ratio of height to width in different buildings or analyze the proportions used in the classic one architecture, such as the proportions of the golden ratio. Students will need to collect data and draw graphs to understand the importance of proportion in architecture.

3. Grocery Store and Units of Measure: Ask students to make a virtual visit to a grocery store where they will have to compare the prices of different products and determine which product is more profitable based on its price-to-quantity ratio. This activity will allow students to apply proportions in a practical way and develop their shopping math skills.

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4. Proportions in art: Introduce students to proportions in art, such as the proportions of the human body in a drawing. Ask them to paint the portrait in the correct proportions. They can also study proportions in the works of famous artists and analyze how proportions affect the composition and reception of a work.
5. Proportions in cartography: The task is to design a map of a specific area, but keeping the proportions. Students will need to consider the scale and ratio of distances on the map to actual distances on the ground. This exercise will help them understand how proportions are used in cartography and how important it is to accurately represent the real world on maps.

$$\begin{aligned}
 x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{-(-10) \pm \sqrt{72}}{2(7)} \quad \text{Positive discriminant} \\
 &= \frac{10 \pm \sqrt{36 \cdot 2}}{14} \\
 &= \frac{10 \pm 6\sqrt{2}}{14} \\
 &= \frac{\cancel{14} (5 \pm 3\sqrt{2})}{\cancel{14}_7} \\
 &= \frac{5 \pm 3\sqrt{2}}{7} \quad \text{Two real solutions}
 \end{aligned}$$

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SYSTEMS OF EQUATIONS

1. Application of systems of equations to real-world problems: Introduce students to various real-world problems that can be modeled using systems of equations. For example, problems related to chemical mix, distribution, or production costs. Ask students to formulate a system of equations and then solve it to find a solution to the problem. This hands-on approach will allow students to see how systems of equations are used to model and solve real-world situations.
2. Interactive Board Game: Create an interactive board game where students have to solve systems of equations to progress through the different levels of the game. Each level may contain a different set of equations, and students will need

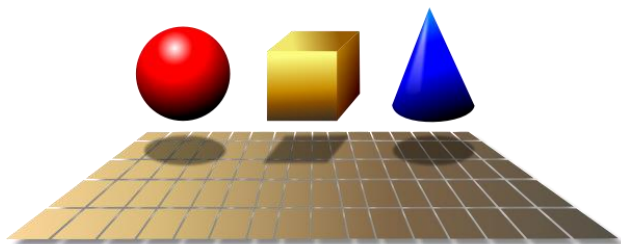
to apply appropriate techniques and skills to find a solution. This method engages students in the subject of systems of equations.

3. Computer simulations: Use computer simulation software that allows students to experiment and see how changes in equation systems affect the results. For example, they can simulate the motions of celestial bodies in space and experiment with various parameters such as body masses and gravitational forces. Students will be able to observe how changes in systems of equations affect the trajectories and behavior of objects.

4. Optimization problems: Give students optimization problems where they have to find the values of variables that maximize or minimize some function. These may be problems related to maximum profit, minimum cost or minimum time to complete the task. Students will have to formulate a system of equations based on a given problem and solve it to find the optimal solution.

5. Research Project: Ask students to conduct a research project that involves the use of systems of equations. For example, they may study relationships between different variables in research, economics, or other fields. Students will need to collect data, analyze it, and formulate a system of equations to find relationships and explain the results. This project-based approach to learning systems of equations develops research skills and logical thinking.

GEOMETRY



1. Study geometric figures with manipulative materials: Provide students with manipulative materials such as geometric blocks, plastic figure models, strings, etc. Ask them to experiment with making different figures and analyze their properties such as number of sides, angles, area and volume. This hands-on approach will allow students to explore geometry interactively and understand its basic concepts.
2. Symmetry around us: Ask students to look for symmetry in the world around them. They can photograph symmetrical objects such as leaves, buildings, carpet

patterns, etc. Then have them analyze the symmetry of these objects, recognize the types of symmetry (central, central, etc.) and create their own symmetrical patterns. This helps students see how symmetry is present in different contexts and develops their observational skills.

3. Geometric constructions: Introduce students to geometric constructions using a compass, ruler and protractor. Ask them to construct various shapes such as triangles, squares, rectangles, etc., using specific parameters, such as side lengths or angle values. Then have them study the properties of these figures and notice the patterns. This activity develops construction skills and the ability to think logically.

4. Geometric puzzles and puzzles: Introduce students to various geometric puzzles and puzzles that require the application of logical thinking and geometric problem-solving skills. For example, puzzles about arranging blocks into certain shapes, problems with determining unknown dimensions of figures, etc. This encourages students to think creatively and solve geometrical difficulties.

5. Geometric Design: Ask students to carry out a geometric project, for example designing their own city or park using different geometric shapes. Students will need to consider proportions, symmetry, spatial arrangement, and other elements of geometry to create a cohesive and interesting space. This design approach develops students' design skills, spatial abilities and creativity.

Motivating students with weaker intellectual potential to learn mathematics



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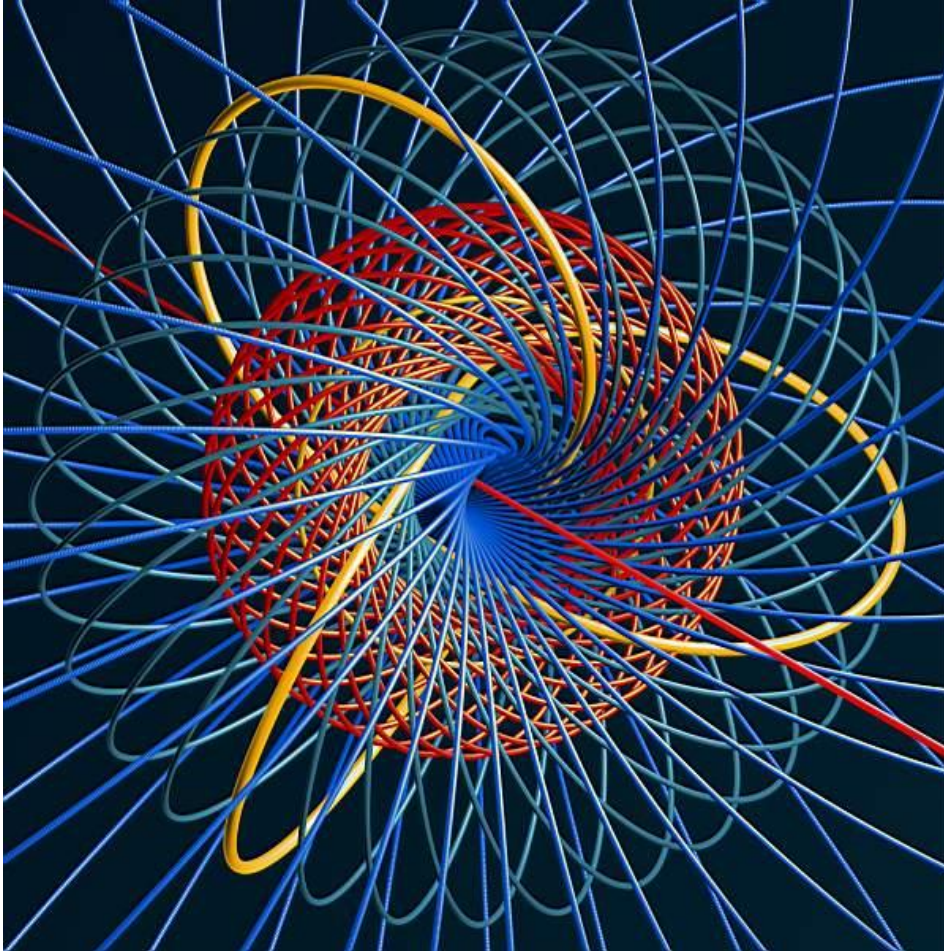
1. Motivating students with lower intellectual potential to learn math can be a challenge, but there are many strategies that can help in the process. Here are some suggestions:
2. Individualize learning: Tailor your teaching approach to each student's needs and abilities. Recognize their strengths and weaknesses in math and develop a personalized learning plan that is tailored to their level.

Students feel more motivated when they feel they are understood and have individual support.

3. Using practical examples: Mathematical examples based on real situations and problems can help students with weaker intellectual potential understand the application of math in everyday life. Show them how math is practical and useful to increase their motivation.
4. Use of multimedia and technology: Use multimedia, math games, apps, interactive online tools, and computer programs to help students learn math in an engaging and interactive way. Such methods may be of interest to students who have difficulty with the traditional approach to mathematics.
5. Create a positive classroom environment: Create an atmosphere of mutual respect, support and a positive approach to learning math. Praise students' progress, recognize their efforts, and celebrate every success. Motivation increases when students feel valued and safe in their environment.
6. Use a variety of teaching methods: Use a variety of teaching methods, such as group work, didactic games, manipulations, hands-on exercises, and visualizations. The diversification of methods allows for variety and adaptation to different learning styles.
7. Intermediate goals and rewards: Set short-term goals for students and reward them for achieving those goals. Objectives should be achievable and specifically related to progress in learning mathematics. Indirect rewards such as praise, certificates, badges or small gifts can further motivate students.
8. Finding interests and passions: Try to find areas of mathematics that are of particular interest to students with lower intellectual potential. It can be geometry, statistics, logical games, etc. Discovering and developing a passion for mathematics can contribute to greater motivation to learn.

It is important to remember that every student is different, so it is important to tailor motivational strategies to the student's individual needs and preferences. Building a positive relationship, support, patience and perseverance are key to effectively motivating students with weaker intellectual potential to learn mathematics.

Here are some visualization ideas that can help you learn algebra:



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Variable manipulation:

Prepare a set of colored variable cards. Put one variable on each card, for example "x" or "y". Ask students to use these cards when solving algebraic equations or expressions. By physically manipulating the variables, they will have a better idea of algebraic operations and understand how variables affect the results.

Modeling linear equations:

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Ask students to create models that represent linear equations. They can use e.g. blocks, colored stripes or other materials. For example, for the equation " $2x + 3y = 8$ ", they might use 2 tiles representing "x", 3 tiles representing "y", and 8 tiles representing the value on the right side of the equation. By manipulating and translating these elements, students will better understand linear equations and their solutions.

Algebraic geometry:

Encourage students to explore the relationship between geometry and algebra. Ask them to choose a geometric figure, such as a rectangle, triangle, or circle, and then calculate the different values associated with that figure, such as perimeter, area, diagonals, etc. Students can use algebraic formulas to calculate these values and see how algebra is related to geometry.

A board game with algebraic problems:

Create a board game for students to solve algebraic problems. Place different problem boxes on the board and students will move around the board, solving algebraic problems to score points or achieve goals. A board game can be both educational and fun, providing an interactive way to learn and reinforce algebraic skills.

Remember that visuals are a powerful tool in learning algebra as they help students visually see abstract concepts and number relationships. They also give the opportunity to experiment and actively participate in the learning process, which can increase students' interest and involvement.

A set of algebra problems for students who are having trouble learning math.

The assignments are designed to provide practical examples and introduce students gradually to the world of algebra. Remember that you can adjust the difficulty and number of tasks depending on the level of students.

1. Algebraic expressions:

a) Rearrange the following algebraic expressions by combining similar terms:

- $2x + 5y - 3x + 2y$
- $4a^2 + 3a + 2a^2 - 5a$

b) Calculate the value of the expressions given the values of the variables:

- $3x + 2y$, for $x = 4$, $y = 6$
- $2a^2 - 3a + 4$, for $a = 2$

2. Linear equations:

a) Solve the linear equations:

- $2x + 3 = 7$
- $4y - 8 = 20$

b) Find the values of x and y in the equation: $2x + 3y = 10$ if $x = 2$.

3. Quadratic equations:

4. a) Solve the quadratic equations:

- $x^2 - 5x + 6 = 0$
- $2y^2 + 3y - 2 = 0$
- b) Find the roots of the quadratic equation: $3x^2 - 7x + 2 = 0$.

5. Proportions and scales:

a) Calculate the missing values in the proportion: $x/4 = 6/12$.

b) The scale of the map is 1 cm : 10 km. If on the map a segment is 8 cm, how many kilometers does it represent?

6. Application of algebraic models:

a) Using the equation $y = mx + c$, plot the function where $m = 2$ and $c = 3$.

b) The formula $A = P(1 + r/n)^{nt}$ describes the residual value of the investment, where A is the residual value, P is the initial value of the investment, r is the annual interest rate, n is the number of compounding periods per year, and t is the number of years. Calculate the residual value of the investment for $P = 5000$, $r = 0.05$, $n = 4$ and $t = 3$.

7. Working with functions:

a) Calculate the function $f(x) = 2x^2 - 3x + 5$ for $x = 2$.

b) Graph the function $g(x) = -x^2 + 4x - 3$.

Remember that it is crucial to answer students, explain the solution steps, and help them understand algebraic concepts. This worksheet is designed to introduce students to algebra and gradually develop their skills in this area.

A guide to solving the tasks:

1. Algebraic expressions: a) To order algebraic expressions, we combine similar terms. We add or subtract expressions with the same variables and degrees.

- $2x + 5y - 3x + 2y = (2x - 3x) + (5y + 2y) = -x + 7y$

- $4a^2 + 3a + 2a^2 - 5a = (4a^2 + 2a^2) + (3a - 5a) = 6a^2 - 2a$

b) To evaluate the expressions given the values of the variables, we substitute the values of the variables into the expressions and perform the calculations.

- For $x = 4$, $y = 6$: $3x + 2y = 3(4) + 2(6) = 12 + 12 = 24$

- For $a = 2$: $2a^2 - 3a + 4 = 2(2^2) - 3(2) + 4 = 2(4) - 6 + 4 = 8 - 6 + 4 = 6$

2. Linear equations: a) To solve linear equations, we move all the terms of the variables to one side and the numbers to the other side to get x or y.

- $2x + 3 = 7$ $2x = 7 - 3$ $2x = 4$ $x = 4/2$ $x = 2$

- $4y - 8 = 20$ $4y = 20 + 8$ $4y = 28$ $y = 28/4$ $y = 7$

b) To find the values of x and y in the equation $2x + 3y = 10$, we substitute $x = 2$ and calculate the value of y: $2(2) + 3y = 10$ $4 + 3y = 10$ $3y = 10 - 4$ $3y = 6$ $y = 6/3$ $y = 2$

3. Quadratic equations: a) To solve quadratic equations, we can use the factoring method or apply quadratic formulas. If the equation cannot be factored, we can use the method of quadratic equations.

• $x^2 - 5x + 6 = 0$ $(x - 2)(x - 3) = 0$ $x = 2$ or $x = 3$

• $2y^2 + 3y - 2 = 0$ $(2y - 1)(y + 2) = 0$ $y = 1/2$ or $y = -2$

b) To find the roots of the quadratic equation $3x^2 - 7x + 2 = 0$, we can solve the equation by factoring or using quadratic formulas.

• $(3x - 1)(x - 2) = 0$ $x = 1/3$ or $x = 2$

4. Proportions and scales:

a) To calculate the missing values in the ratio $x/4 = 6/12$, we can use the rule of equality of cross products. $x/4 = 6/12$ $12x = 4 * 6$ $12x = 24$ $x = 24/12$ $x = 2$

b) The scale of the map is 1 cm : 10 km. If the segment on the map is 8 cm, to calculate how many kilometers it represents, we need to multiply the length of the segment on the map by the scale. $8 \text{ cm} * 10 \text{ km/cm} = 80 \text{ km}$

5. Application of algebraic models:

a) Using the equation $y = mx + c$, where $m = 2$ and $c = 3$, we can plot the function. We select different x values, compute the corresponding y values, and create points on the graph that we connect with a line. Example points: (0, 3), (1, 5), (2, 7), (3, 9), etc.

b) The formula $A = P(1 + y/n)^{nt}$ describes the residual value of the investment. Substituting the values of $P = 5000$, $r = 0.05$, $n = 4$ and $t = 3$ into the formula, we can calculate the final value of A . $A = 5000(1 + 0.05/4)^{(4*3)}$ $A = 5000(1 + 0.0125)^{(12)}$ $A = 5000(1.0125)^{(12)}$ $A \approx 5000 * 1.159$ $A \approx 5795$

6. Working with functions:

a) To calculate the value of the function $f(x) = 2x^2 - 3x + 5$ for $x = 2$, we substitute $x = 2$ into the equation and do the calculations. $f(2) = 2(2^2) - 3(2) + 5$ $f(2) = 2(4) - 6 + 5$ $f(2) = 8 - 6 + 5$ $f(2) = 7$

b) To plot the function $g(x) = -x^2 + 4x - 3$, we select different x values, compute the corresponding y values, and create points on the graph that we connect with a line. Example points: (-2, -15), (-1, -8), (0, -3), (1, 0), etc.

With this guide, students will have clear guidance for solving algebra problems and will be able to work on increasing their skills in this area of math. Remember to give them help and explanations if they encounter difficulties.

Some visualizations that can help you learn proportions:



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Proportional ruler set: Prepare several rulers of different lengths (e.g. 1 cm, 2 cm, 3 cm, etc.). You can use a ruler or other drawing tools. Put them side by side on a piece of paper and see how they scale. This will help you see how the length proportion varies with size.

Draw geometric figures: Select a geometric figure, such as a triangle, square, rectangle, or circle. Then draw several variants of the same figure, but in different proportions. For example, draw triangles with different side lengths and compare their proportions. This will help you understand how changing the proportions affects the appearance of the figure.

Folding the paper: Take a piece of paper and fold it in different proportions. You can start with a rectangle and fold it in half, third, fourth, etc. This will help you see how the ratio of length and width changes depending on how you fold it.

Comparing items: Collect various items of different sizes, such as fruits, pens, books, etc. Place them side by side and compare their sizes. You can also use boxes to create different proportions in three dimensions.

Drawing body proportions: If you are interested in learning the proportions of the human body, you can use different drawing techniques such as division of proportions on the head to create a proportion visualization. There are many online tutorials that show you how to draw the proportions of the human body, taking into account the different proportions between the body parts.

Remember that learning proportions takes practice and observation. Use these visualizations as a tool to better understand how proportions affect different objects and drawings.

A set of proportion worksheets for students who have trouble learning math:

Excercise 1:

In the store they sell 3 bottles of juice for PLN 6. How much is 7 bottles of juice?

Exercise 2:

On the map, the distance between the two cities is 9 centimeters. If the map scale is 1 centimeter = 50 kilometers, what is the real distance between these cities?

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Exercise 3:

On the soccer field, team A has 18 players and team B has 15 players. How many players does team C have if the ratio between the number of players from team A and team B is 3:5?

Exercise 4:

On the plan of the house, the proportion between the actual length of the room and its length on the plan is 1:50. If the length of the room on the plan is 8 centimeters, what is the actual length of the room?

Exercise 5:

To bake 24 cookies, you need 3 cups of flour. How many cups of flour will be needed to bake 40 cookies?

Exercise 6:

The speed of sound in air is about 340 meters per second. How long will it take the sound to travel 1.5 km?

Exercise 7:

In the playground, 12 children play on 3 swings. How many swings will be needed if 24 children want to play?

Exercise 8:

In a triangle ABC, the ratio of side AB to side BC is 3:4, and side BC is 12 centimeters. How many centimeters is side AB?

Exercise 9:

Two cars are moving simultaneously from two different points towards each other. Car A is traveling at 60 km/h and car B is traveling at 80 km/h. If the distance between them is 240 kilometers, how many hours will it take before they meet?

Exercise 10:

At the prom, $\frac{2}{5}$ of the participants are boys and the rest are girls. If the number of boys is 60, how many attendees are at the prom?

A guide to solving a set of proportions problems:

Excercise 1:

Use the ratio to find the price of 1 bottle of juice. Divide PLN 6 by 3 bottles to get the price of one bottle ($\text{PLN } 6 / 3 = \text{PLN } 2$). Then multiply the price of one bottle (PLN 2) by the number of bottles (7) to find the cost of 7 bottles of juice ($\text{PLN } 2 * 7 = \text{PLN } 14$).

Answer: 7 bottles of juice cost PLN 14.

Exercise 2:

To find the real distance between cities, multiply the distance on the map (9 centimeters) by the scale of the map (50 kilometers per centimeter). As a result, you will get the real distance between cities.

Calculation: $9 \text{ cm} * 50 \text{ km/cm} = 450 \text{ km}$

Answer: The actual distance between the cities is 450 kilometers.

Exercise 3:

To find the number of players in team C, use the proportion. The ratio between the number of players from team A and team B is 3:5. First calculate the common denominator: $3 + 5 = 8$. Then divide the number of players by the common denominator and multiply by 8 to find the number of players for team C.

Calculation: $(18/8) * 8 = 18$

Answer: Team C has 18 players.

Exercise 4:

To find the actual length of the room, divide the length of the room on the plan (8 centimeters) by the proportion (1:50). Multiply the result by 50 to get the actual length of the room.

Calculation: $8 \text{ cm} / 50 = 0.16 \text{ cm}$

$0.16\text{cm} * 50 = 8\text{cm}$

Answer: The actual length of the room is 8 centimeters.

Exercise 5:

To find the number of cups of flour needed to bake 40 cookies, use the ratio. Find the proportion between the number of cookies and the amount of flour. You know that 24 cookies require 3 cups of flour. Divide the number of cookies by 24 and multiply by 3 to find the number of cups of flour needed to bake 40 cookies.

Calculation: $(40/24) * 3 = 5$

Answer: You need 5 cups of flour to bake 40 cookies.

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Exercise 6:

To calculate the time it takes sound to travel a distance, divide the distance (1.5 kilometers) by the speed of sound (340 meters per second).

Calculation: $1.5 \text{ km} / 0.34 \text{ km/s} = 4.41 \text{ s}$

Answer: The sound will take approximately 4.41 seconds to travel 1.5 kilometers.

Exercise 7:

To find the number of swings needed for 24 children, use the ratio. The ratio of the number of children to the number of swings is the same for both cases. Divide the number of children by the number of swings for 12 children, then multiply the result by 24 children to find the number of swings needed for 24 children.

Calculation: $(24/12) * 3 = 6$

Answer: It takes 6 swings for 24 children to play.

Exercise 8:

The ratio of the length of the side AB to the side BC is 3:4, and the length of the side BC is 12 centimeters. To find the length of side AB, divide the length of side BC by 4, then multiply the result by 3.

Calculation: $(12 \text{ cm} / 4) * 3 = 9 \text{ cm}$

Answer: Side AB is 9 centimeters long.

Exercise 9:

To find the time before two cars meet, divide the distance between them (240 km) by the sum of their speeds (60 km/h + 80 km/h).

Calculation: $240 \text{ km} / (60 \text{ km/h} + 80 \text{ km/h}) = 2 \text{ hours}$

Answer: Two cars will meet after 2 hours.

Exercise 10:

To find the number of participants at the prom, use the proportion. You know that $\frac{2}{5}$ of the participants are boys, and the number of boys is 60. Divide the number of boys by $\frac{2}{5}$, then multiply the result by 5.

Here is a visualization proposal for teaching systems of equations:

$$\begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} = -\frac{1}{m} R_t K_t R_t^T \dot{\zeta} - G \quad (15)$$

$$\begin{bmatrix} f_\phi \\ f_\theta \\ f_\psi \end{bmatrix} = -(I_T R_r)^{-1} \left[I_T \left(\frac{\partial R_r}{\partial \phi} \dot{\phi} + \frac{\partial R_r}{\partial \theta} \dot{\theta} \right) \dot{\eta} - K_r R_r \dot{\eta} - (R_r \dot{\eta}) \times (I_T R_r \dot{\eta}) \right] + \begin{bmatrix} \frac{c}{I_z} C_\phi T_\theta \sum_{i=1}^4 (-1)^{i+1} F_i \\ -\frac{c}{I_z} S_\phi \sum_{i=1}^4 (-1)^{i+1} F_i \\ \frac{d}{I_y} S_\phi S e_\theta (F_3 - F_1) \end{bmatrix}$$

${}^1T_\theta$ and $S e_{(\cdot)}$ are respectively the abbreviations of $\tan(\cdot)$ and $\frac{1}{\cos(\cdot)}$

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Equation board: The teacher can prepare a large board on which the equations from the system of equations will be drawn. Each equation should be placed on a separate line. For example:

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$$2x + 3y = 10$$

$$x - y = 2$$

Variable Blocks: Use blocks of different colors or shapes to represent variables. For example, let the blue block represent the variable "x" and the yellow block denote the variable "y". By moving these blocks along the equations on the board, students will be able to see how the variables affect the equations.

Arrows and Labels: Use arrows to indicate assigned coefficients on variables. For example, an arrow from a "2x" block may indicate a "2" factor, and an arrow from a "3y" block may indicate a "3" factor. Additionally, mark equations as "1", "2", "3" etc. to make it easier to find specific equations during explanations.

Block Manipulation: Have students manipulate the blocks of variables by moving them according to the math they want to apply to the equations. For example, if the equation requires adding 2 to "x", students can move the "x" block 2 units to the right. This will help them see how this operation affects the equation.

Marking Common Solutions: Highlight the specific position of the blocks that represent the solution values of a system of equations. You can do this with circles or squares around the blocks. Show students how the position of the blocks together means solving a system of equations.

Graphs: Plot equations on the Cartesian axis, where the "x" axis represents one variable and the "y" axis represents the other variable. Show students how graph intersections represent solutions to a system of equations.

Interactive online tools: Use online interactive tools that allow students to experiment with systems of equations. There are many apps and websites that allow you to create and manipulate equations, which can help students better understand the concept of systems of equations.

A set of systems of equations activities for students with math learning disabilities:

1. Solve the following system of equations using the substitution method:

$$2x + 3y = 10$$

$$x - y = 2$$

2. Solve the following system of equations by elimination:

$$3x + 2y = 8$$

$$2x - y = 1$$

3. Solve the following system of equations graphically:

$$y = 2x - 3$$

$$y = -x + 5$$

4. Solve the following system of equations using the substitution method:

$$4x + 5y = 17$$

$$2x + y = 7$$

5. Solve the following system of equations by elimination:

$$3x + 4y = 11$$

$$6x + 8y = 22$$

6. Solve the following system of equations graphically:

$$2x + 3y = 6$$

$$4x - y = 8$$

7. Solve the following system of equations using the substitution method:

$$2x - y = 5$$

$$3x + 2y = 8$$

8. Solve the following system of equations by elimination:

$$5x + 3y = 12$$

$$4x + 2y = 10$$

9. Solve the following system of equations graphically:

$$y = -2x + 4$$

$$y = 3x - 1$$

10. Solve the following system of equations using the substitution method:

$$3x - 2y = 7$$

$$x + 4y = 5$$

A guide to solving problems with systems of equations:

Solving the system of equations by substitution method:

a) Look at the first equation: $2x + 3y = 10$. We can solve this equation by expressing one variable (x or y) from the second equation.

b) Look at the second equation: $x - y = 2$. We can express x from this equation:
 $x = y + 2$.

c) Substitute this expression for x in the first equation: $2(y + 2) + 3y = 10$.

- d) Expand the bracket: $2y + 4 + 3y = 10$.
- e) Match the terms with y : $5y + 4 = 10$.
- f) Subtract 4 from both sides of the equation: $5y = 6$.
- g) Divide both sides by 5: $y = 6/5 = 1.2$.
- h) Now, to find x , substitute the value of y into the second equation: $x = 1.2 + 2 = 3.2$.
- i) The solution to this system of equations is $x = 3.2$ and $y = 1.2$.

Solving a system of equations by elimination:

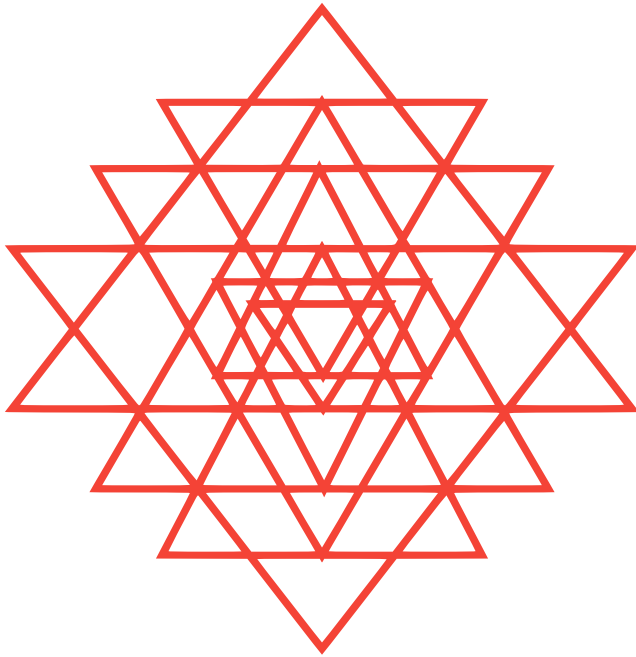
- a) Look at both equations and decide which variable you want to get rid of by elimination. In this case, we will eliminate the variable x .
- b) Multiply the first equation by 2 and the second equation by 3 to match the coefficients at x and bring them to opposite signs: $4x + 6y = 20$ and $6x - 3y = 3$.
- c) Add these equations: $(4x + 6y) + (6x - 3y) = 20 + 3$.
- d) Reduce to the form: $10x + 3y = 23$.
- e) Solve this equation to calculate the value of one of the variables. For example, expressing x from this equation: $x = (23 - 3y) / 10$.
- f) Substitute this expression for x into one of the initial equations and solve it to find the value of the second variable.
- g) In this case, substituting $x = (23 - 3y) / 10$ into the first equation, we get: $2((23 - 3y) / 10) + 3y = 10$.
- h) Solve this equation to calculate the y -value. Then, substitute this y -value into one of the equations to calculate the x -value.
- i) The solution to this system of equations is $x = 3.2$ and $y = 1.2$.

Solving the system of equations graphically:

- a) Draw the graphs of both equations on the Cartesian system, labeling the x-axis and the y-axis.
- b) Find the intersection of the graphs. This is a solution to a system of equations.
- c) In this case, the intersection of the graphs represents $x = 3.2$ and $y = 1.2$, which is a solution to the system of equations.

Some visualizations that can help you learn geometry:

Visualization of geometric figures:



Generate an interactive visualization of various geometric figures such as triangles, rectangles, squares, circles, etc. The visualization should show the dimensions and proportions of the figures so that students can see their characteristics and relationships.

Interactive scatter plot:

Plot various points in space on a scatter plot, and students can manipulate them by moving, scaling, or rotating them. This will help them understand concepts such as point coordinates, the Cartesian system, and distances between points.

Angle Simulation:

Create a visualization that allows you to manipulate angles. Students can change the values of the angles and observe how their magnitudes change and how they affect the relationships between other angles.

Visualization of geometry theorems:

Visualize geometric theorems such as the Pythagorean theorem, Thales theorem, and the law of sines. Through animations and interaction, students will be able to see how these theorems work with examples of specific geometric figures.

Animation of geometric constructions:

Create animations that demonstrate the process of constructing various geometric figures, such as parallel lines, rectangles, and axial symmetry. This will help students visually understand the construction steps and the relationships between the different elements.

A set of geometry tasks for students having trouble learning math:

1. Task: Find the area of a rectangle with a side length of 5 cm and a side width of 8 cm.

Question: Find the perimeter of an equilateral triangle whose side is 6 cm.

Problem: Calculate the volume of a cube with an edge length of 4 cm.

2. Task: Calculate the circumference of a circle with a radius of 5 cm.

Question: Find the length of the diagonal of a rectangle with sides 9 cm and 12 cm.

3. Task: Calculate the area of a trapezoid with bases 5 cm and 8 cm long and a height of 6 cm.

Problem: Find the area of a right triangle whose legs are 3 cm and 4 cm.

4. Task: Calculate the volume of a cone with base radius 2 cm and height 6 cm.
5. Task: Calculate the perimeter of a square with sides of length 7 cm.

Question: Find the area of a triangle with base 10 cm and height 8 cm.

It's a good idea to provide students with drawings or diagrams to help them visualize and understand the problem. Encourage students to use appropriate formulas and equations to solve geometric problems.

Remember that as a teacher, you can adapt the difficulty of these tasks to the level and skills of your students by adjusting the numerical values or adding additional conditions. It is important to provide adequate explanation and guidance in the event of student difficulties.

A guide to solving a set of geometry problems:

1. Task: Find the area of a rectangle with a side length of 5 cm and a side width of 8 cm.

The area of a rectangle is calculated by multiplying the side length by the width. In this case, the area of the rectangle = 5 cm * 8 cm = 40 cm².

Question: Find the perimeter of an equilateral triangle whose side is 6 cm.

An equilateral triangle has all sides of equal length. The perimeter of an equilateral triangle is calculated by multiplying the length of one side by 3. In this case, the perimeter of an equilateral triangle = 6 cm * 3 = 18 cm.

Problem: Calculate the volume of a cube with an edge length of 4 cm.

The volume of a cube is calculated by raising the length of the edges to the third power. In this case, the volume of the cube = 4 cm * 4 cm * 4 cm = 64 cm³.

2. Task: Calculate the circumference of a circle with a radius of 5 cm.

The circumference of a circle is calculated by multiplying the radius by 2π (pi). In this case, the circumference of a circle = 5 cm * $2\pi \approx 31.42$ cm.

Question: Find the length of the diagonal of a rectangle with sides 9 cm and 12 cm.

The length of the diagonal of a rectangle is calculated using the Pythagorean theorem. In this case, the length of the diagonal = $\sqrt{(9 \text{ cm}^2 + 12 \text{ cm}^2)} \approx 15$ cm.

3. Task: Calculate the area of a trapezoid with bases 5 cm and 8 cm long and a height of 6 cm.

The area of a trapezoid is calculated by multiplying the sum of the lengths of the bases by the height and then dividing by 2. In this case, the area of the trapezoid = (5 cm + 8 cm) * 6 cm / 2 = 39 cm².

Problem: Find the area of a right triangle whose legs are 3 cm and 4 cm.

The area of a right triangle is calculated by multiplying the length of one side by the length of the other side, then dividing by 2. In this case, the area of a right triangle = (3 cm * 4 cm) / 2 = 6 cm².

4. Task: Calculate the volume of a cone with base radius 2 cm and height 6 cm.

The volume of the cone is calculated by the formula $V = (1/3) * \pi * r^2 * h$, where r is the radius of the base, h is the height of the cone. In this case, the volume of the cone = $(1/3) * \pi * (2 \text{ cm})^2 * 6 \text{ cm} \approx 25.13 \text{ cm}^3$.

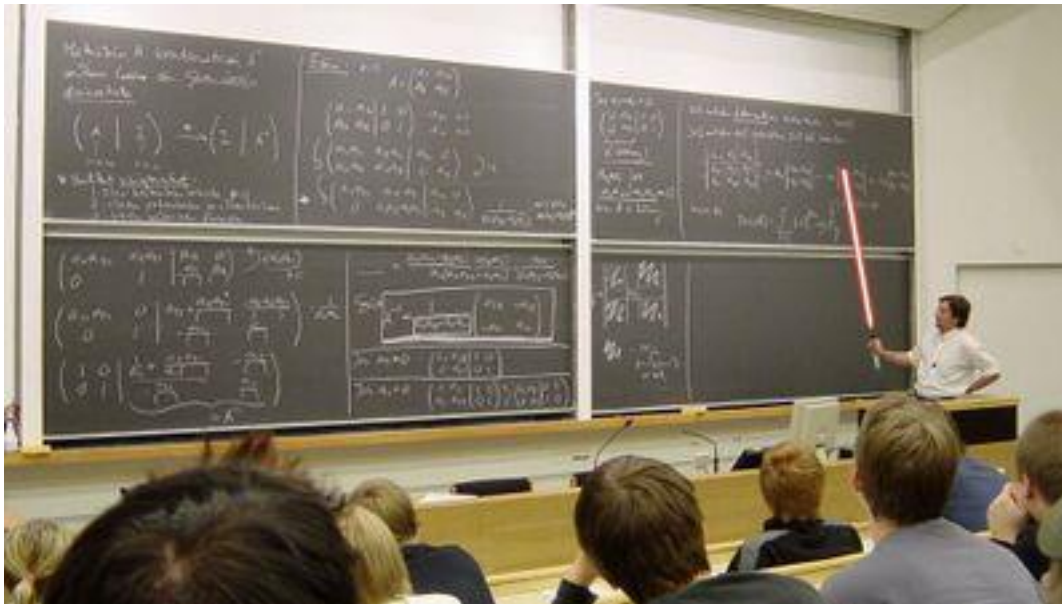
5. Task: Calculate the perimeter of a square with sides of length 7 cm.

The perimeter of a square is calculated by multiplying the length of one side by 4. In this case, the perimeter of the square = $7 \text{ cm} * 4 = 28 \text{ cm}$.

Question: Find the area of a triangle with base 10 cm and height 8 cm.

The area of a triangle is calculated by multiplying the length of the base by the height and then dividing by 2. In this case, the area of the triangle = $(10 \text{ cm} * 8 \text{ cm}) / 2 = 40 \text{ cm}^2$.

Remember that it is important to understand the formulas and equations used in each task, as well as the accuracy of calculations and units of measurement. Encourage students to check their answers and the accuracy of their results.



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Mathematics teaching guide for gifted students

Teaching mathematics to gifted students can be a challenge, but also an opportunity to develop your skills and passion for this subject. Below is a guide that can help you effectively teach mathematics to gifted students:

Skill Level Diagnosis:

Make an accurate diagnosis of each student's math skill level. Discover their strengths and weaknesses and areas where they have more interest and potential. Thanks to this, you will be able to tailor your approach and materials to their individual needs.

Use complex tasks:

Gifted students often look for challenges. Therefore, it is worth providing them with access to complex and advanced mathematical tasks. You can use open problems, everyday math problems or math logic puzzles. It is important to encourage them to think analytically and solve problems creatively.

Individualization of learning:

Give gifted students the opportunity to learn at a pace that suits them best. Provide them with additional materials, resources, and activities that are relevant to their skills and interests. You may also consider one-on-one consultations where they will have the opportunity to explore more advanced topics.

Apply technology:

Take advantage of technological tools such as interactive applications, mathematical visualization programs, simulations and development tools. Thanks to them, talented students will be able to experiment, discover new mathematical concepts and develop their problem-solving skills.

Use math projects:

Encourage gifted students to work on math projects that involve research, data analysis, and reasoning. Projects can cover various areas of mathematics, such as statistics, geometry, algebra or data analysis. This will enable them to put their skills into practice and develop not only math skills but also creative thinking and presentation skills.

Stimulate discussion and collaboration:

Organize discussions, presentations and workshops where talented students have the opportunity to share their ideas, solutions and math strategies. Working together in groups will allow them to exchange ideas and learn from each other.

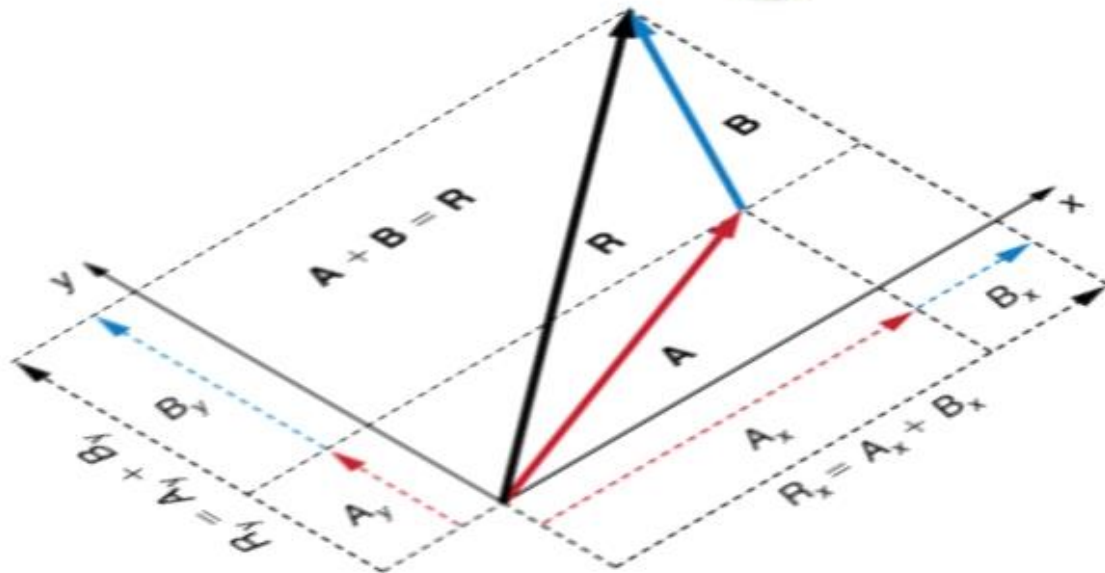
Support their passion:

Be a support for the passions and interests of mathematically gifted students. Give them the opportunity to explore specific topics, participate in math Olympiads, competitions or other math events. This will allow them to develop their abilities and ambitions.

No matter how capable students are, it is important to develop their interest in math, stimulate their analytical thinking and creative problem solving. By giving

them the opportunity to explore and discover, you open the door to their mathematical development and success.

Vector Algebra



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A Guide to Solving Difficult Algebra Problems:

When solving complex algebraic expressions, pay attention to similar terms and use the properties of algebraic operations, such as combining and simplifying expressions.

When solving linear equations, think carefully about what steps you take to avoid mistakes. Carefully expand and simplify the equations, then find the x values that satisfy the equation.

When solving quadratic equations, use appropriate methods such as the quadratic method, factorization, square's complement, or Viète's formulas to find the roots of the equation.

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When solving inequalities, remember to change the sign correctly and take into account domain constraints that may affect the solution.

When working with functions, calculate function values for given values of x , solve equations and inequalities, and find zeros and the domain of a function.

For matrix operations, carefully perform matrix multiplication, addition and subtraction of matrix elements, and solving systems of matrix equations.

When calculating logarithms, use the properties of logarithms, such as the logarithm of the sum and difference, the logarithm of the product and the quotient, and use the appropriate properties of logarithmic equations.

When calculating operations with complex numbers, remember the properties of addition, subtraction, multiplication, and division of complex numbers.

When solving exponential equations, use appropriate methods such as substitution, logarithms, and properties of exponential functions to find the value of x that satisfies the equation.

Remember to analyze the problem, use the right tools and techniques, and check the results to make sure you get the right solution.

I encourage students who are able to raise questions, ask additional problems and experiment with different methods of problem solving. Practice, determination and curiosity are key to improving your math skills.

A set of challenging algebra problems:

Algebraic expressions:

- a) Rearrange and simplify the expression: $(2x^3 + 3x^2 - x + 4) - (x^3 - 2x^2 + 5x - 3)$
- b) Expand and simplify the expression: $(3x - 2y)^2$

Linear equations:

- a) Solve the equation: $2(3x + 4) - 5(2x - 1) = 3(2x + 1) - 4(x - 3)$
- b) Solve the equation: $|2x - 5| = 7$

Quadratic equations:

- a) Solve the equation: $x^2 + 5x + 6 = 0$
- b) Solve the equation: $2x^2 + 3x - 4 = 0$

Inequalities:

- a) Solve the inequality: $2x - 5 < 3x + 2$
- b) Solve the inequality: $(x - 3)(x + 2) > 0$

Functions:

- a) Calculate the function $f(x) = \frac{x^2 + 3x - 2}{x - 1}$ for $x = 4$
- b) Find the zeros of $g(x) = x^3 - 2x^2 + x - 3$

Matrices:

- a) Calculate the matrix product $A = \begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & -2 \\ 1 & 3 \end{bmatrix}$
- b) Solve the system of matrix equations: $\begin{bmatrix} 3 & -2 \\ 2 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \end{bmatrix}$

Logarithms:

- a) Solve the equation: $\log(x + 2) + \log(x - 1) = 2$
- b) Evaluate the expression: $\log(\sqrt{3}) + \log(\sqrt{27})$

Complex numbers:

- a) Find the roots of the complex quadratic equation: $z^2 + 4z + 5 = 0$
- b) Evaluate the expression: $(2 + 3i)(1 - 2i)$

The concept of a compound function:

- a) Calculate $h(x) = \sqrt{2x + 3}$ if $f(x) = x^2 - 4$ and $g(x) = 2x + 1$
- b) Find the domain of the composite function: $f(g(x))$ if $f(x) = \sqrt{x}$ and $g(x) = 2x - 3$

Solutions of exponential equations:

- a) Solve the equation: $2^x - 3 = 5$
- b) Solve the equation: $e^x - 2e^{-x} = 3$

Answers to tasks:

Algebraic expressions:

- a) $(2x^3 + 3x^2 - x + 4) - (x^3 - 2x^2 + 5x - 3) = x^3 + 5x^2 - 6x + 7$
- b) $(3x - 2y)^2 = 9x^2 - 12xy + 4y^2$

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Linear equations:

- a) Solving the equation: $x = -1/3$
- b) Solving the equation: $x = 6, x = -1$

Quadratic equations:

- a) Solving the equation: $x = -2, x = -3$
- b) Solving the equation: $x = 1/2, x = -4/3$

Inequalities:

- a) Solution of the inequality: $x < 7$
- b) Solution of the inequality: $x < -2$ or $x > 3$

Functions:

- a) The value of $f(x) = (x^2 + 3x - 2)/(x - 1)$ for $x = 4$ is 21.
- b) The zeros of $g(x) = x^3 - 2x^2 + x - 3$ are $x = -1, x = 1, x = 3$.

Matrices:

- a) The product of matrices A and B is: $[7 \ -4; -19 \ 2]$
- b) Solving the system of matrix equations $[3 \ -2; 2 \ 1] * [x; y] = [7; 4]$ then $x = 1, y = 2$.

Logarithms:

- a) Solving the equation: $x = 3, x = 0$
- b) The value of the expression: $\log(\sqrt{3}) + \log(\sqrt{27}) = 1/2 + 3/2 = 2$

Complex numbers:

- a) Roots of a complex quadratic equation: $z = -2 - i$, $z = -2 + i$
- b) The value of the expression: $(2 + 3i)(1 - 2i) = 8 + i$

The concept of a compound function:

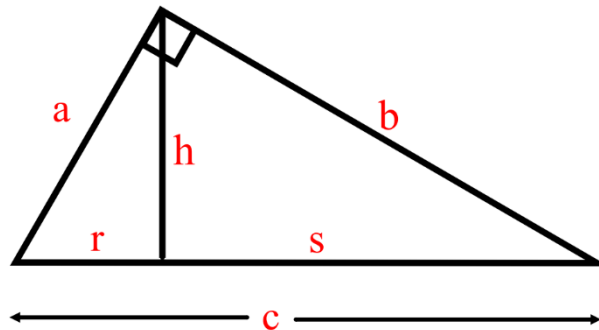
- a) The value of $h(x) = \sqrt{2x + 3}$ for $x = 4$ is $\sqrt{11}$.
- b) The domain of the composite function $f(g(x))$ is $x > 3/2$.

Solutions of exponential equations:

- a) Solving the equation: $x = 4$
- b) Solving the equation: $x = 1.316$

A guide to solving difficult proportion problems

Complete the proportion



$$\frac{r}{h} = \frac{a}{\square}$$

$$\frac{h}{a} = \frac{\square}{b}$$

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Proportions are an important topic in math, and solving them can be difficult for some students. Below I present a guide that will help you in solving more difficult proportion problems.

Task: Calculate the value of x in the proportion: $3/5 = x/15$

Solution:

Use the property of proportion, which says that the cross product is equal to: $a/b = c/d$, so $ad = bc$.

Substitute the data from the proportion: $3 * 15 = 5 * x$.

Calculate the expression: $45 = 5x$.

Divide both sides of the equation by 5: $x = 9$.

Answer: $x = 9$.

Task: Calculate the value of y in the proportion: $4/9 = 12/y$

Solution:

Use the proportion property: $a/b = c/d$, $ad = bc$.

Substitute the data from the proportion: $4 * y = 9 * 12$.

Calculate the value of the expression: $4y = 108$.

Divide both sides of the equation by 4: $y = 27$.

Answer: $y = 27$.

Task: Fill in the missing values in the proportion: $8/12 = x/18$

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Solution:

Find the cross product: $8 * 18 = 12 * x$.

Calculate the expression: $144 = 12x$.

Divide both sides of the equation by 12: $x = 12$.

Answer: $x = 12$.

Task: Find the value of x in the proportion: $2/3 = 10/x$

Solution:

Use the proportion property: $a/b = c/d$, $ad = bc$.

Substitute the data from the proportion: $2 * x = 3 * 10$.

Calculate the expression: $2x = 30$.

Divide both sides of the equation by 2: $x = 15$.

Answer: $x = 15$.

Task: Calculate the value of y in the proportion: $5/8 = y/20$

Solution:

Apply the proportion property: $a/b = c/d$, $ad = bc$.

Substitute the data from the proportion: $5 * 20 = 8 * y$.

Calculate the expression: $100 = 8y$.

Divide both sides of the equation by 8: $y = 12.5$.

Answer: $y = 12.5$.

Task: Fill in the missing values in the proportion: $x/4 = 6/9$

Solution:

Find the cross product: $9 * x = 6 * 4$.

Evaluate the expression: $9x = 24$.

Divide both sides of the equation by 9: $x = 24/9$.

Answer: $x = 8/3$.

Remember that solving proportion problems requires the ability to apply the properties of proportion and the ability to solve equations. It is also important to maintain equivalence in each solving step. Practice regularly and solving proportion problems will become easier.

A set of tricky proportion tasks:

1: Calculate the value of x in the proportion: $(3x + 4)/(2x - 1) = 5/3$

2: Fill in the missing values in the proportion: $(x + 3)/(2x + 5) = 4/7$

3: Calculate the value of y in the proportion: $(2y + 1)/(3y - 2) = 6/5$

4: Fill in the missing values in the proportion: $(4x + 5)/(6x - 3) = 2/9$

5: Calculate the value of x in the proportion: $(5x - 3)/(4x + 7) = 3/2$

6: Fill in the missing values in the proportion: $(3y - 2)/(5y + 1) = 7/4$

7: Calculate the value of x in the proportion: $(2x - 1)/(3x + 2) = 5/6$

8: Fill in the missing values in the proportion: $(7y + 4)/(6y - 5) = 9/8$

9: Calculate the value of x in the proportion: $(4x + 3)/(2x - 1) = 7/5$

10: Fill in the missing values in the proportion: $(5y - 2)/(3y + 1) = 8/9$

Here are the answers to a set of difficult proportion tasks:

1: $x = 9/7$

2: $x = 2/3$

3: $y = -2/7$

4: $x = -3/2$

5: $x = -13/7$

6: $y = 2/3$

7: $x = 5/4$

8: $r = 39/59$

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$$9: x = 5/6$$

$$10: y = 23/37$$

Please note that the answers above have been calculated based on a given set of tasks. Make sure you understand the process of solving each problem and check your calculations carefully.

A guide to solving difficult problems with systems of equations:

$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\&= \frac{-(-10) \pm \sqrt{72}}{2(7)} && \text{Positive discriminant} \\&= \frac{10 \pm \sqrt{36 \cdot 2}}{14} \\&= \frac{10 \pm 6\sqrt{2}}{14} \\&= \frac{\cancel{2} (5 \pm 3\sqrt{2})}{\cancel{14}_7} \\&= \frac{5 \pm 3\sqrt{2}}{7} && \text{Two real solutions}\end{aligned}$$

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Difficult tasks with systems of equations require a meticulous approach and the use of appropriate methods. For complex systems of equations, it is useful to use elimination or substitution to find solutions.

Make sure that the system of equations is spelled correctly and that all words are in the correct order.

Choose the appropriate method for solving the system of equations, e.g.:

The elimination method manipulates equations to eliminate one variable and produce a single-variable equation. Then calculate the value of this variable and substitute it into the second equation to calculate the value of the second variable.

The substitution method involves solving one equation for one variable and then substituting the resulting value of that variable into another equation to calculate the value of the other variable.

Do your calculations carefully to avoid arithmetic errors. Make sure you do the right operations on both sides of the equation to keep your balance.

Test your solution by substituting the values of the variables into the original equations. Make sure you get true equalities.

If you get a contradiction, i.e. equation $0 = 0$ or false, then the system of equations is inconsistent and has no solution. If we get the equality $0 = \text{non-zero number}$, it means that the system of equations is infinite and has infinitely many solutions.

Don't forget to represent your answers as pairs or triples of variable values, depending on the number of unknowns in the system of equations.

Remember that solving difficult problems with systems of equations requires practice and an understanding of different solving methods. Practice regularly to gain confidence in solving these types of tasks.

Here is a set of complex problems with systems of equations:

1. Solve the system of equations:

$$\{ 2x + 3y - z = 7$$

$$\{ x - 2y + 4z = 1$$

$$\{ 3x + 2y - 3z = 5$$

2. Solve the system of equations:

$$\{ 2x + 3y - 5z = 10$$

$$\{ 3x - 2y + 4z = 5$$

$$\{ x + 4y - z = 7$$

3. Solve the system of equations:

$$\{ x^2 + y^2 = 10$$

$$\{ x - y = 1$$

4. Solve the system of equations:

$$\{ \log(x + y) = 2$$

$$\{ 2x - 3y = 1$$

5. Solve the system of equations:

$$\{ \sqrt{x} + \sqrt{y} = 5$$

$$\{ x^2 - y^2 = 3$$

6. Solve the system of equations:

$$\{ e^x + 2y = 10$$

$$\{ 3x - 2e^y = 5$$

7. Solve the system of equations:

$$\{ \sin(x) + \cos(y) = 1$$

$$\{ 2x + y = \pi/4$$

8. Solve the system of equations:

$$\{ x^2 + y^2 = 25$$

$$\{ x^3 + y^3 = 72$$

9. Solve the system of equations:

$$\{ \sqrt{x} + \sqrt{y} = 5$$

$$\{ x^2 - 3y = 7$$

10. Solve the system of equations:

$$\{ \log(x + y) = 3$$

$$\{ x^2 - y^2 = 5$$

Remember that solving complex problems with systems of equations may require the use of various methods, such as Gaussian elimination, substitution, or the graphical method. Do the calculations carefully and check the results to make sure you get the correct solutions.

A set of answers to problems with systems of equations:

1. Solving the system of equations:

$$x = 1$$

$$y = 2$$

$$z = 3$$

2. Solving the system of equations:

$$x = 2$$

$$y = -1$$

$$z = 3$$

3. Solving the system of equations:

$$x = 1 + \sqrt{9}$$

$$y = 1 - \sqrt{9}$$

4. Solving the system of equations:

$$x = 4$$

$$y = 3$$

5. Solving the system of equations:

$$x = 2$$

$$y = 3$$

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6. Solving the system of equations:

$$x = 2$$

$$y = 3$$

7. Solving the system of equations:

$$x = \pi/4 - y$$

$$y = \pi/4 - 2*(\pi/4 - y)$$

8. Solving the system of equations:

$$x = 3$$

$$y = 4$$

9. Solving the system of equations:

$$x = 4$$

$$y = 1$$

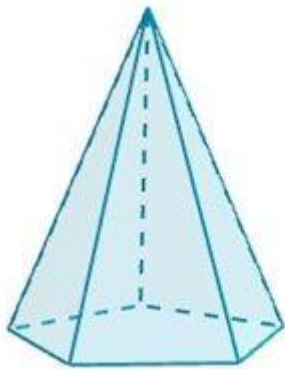
10. Solving the system of equations:

$$x = 10$$

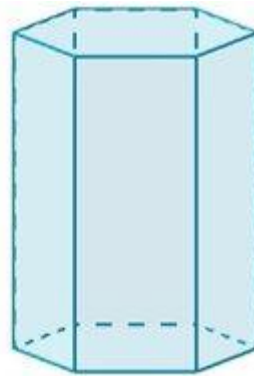
$$y = 10 - x$$

Be sure to double check your calculations and make sure you get the same results.

A guide to solving difficult geometry problems:



Pirámide



Prisma

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Solving problems with the Pythagorean theorem:

- a) Check if the Pythagorean theorem is true in a triangle, i.e. if the sum of the squares of the lengths of the two shorter sides is equal to the square of the length of the longest side.
- b) If so, calculate the length of the missing side using the Pythagorean theorem: $a^2 + b^2 = c^2$, where a and b are the lengths of the known sides and c is the length of the missing side.

Solving tasks from the similarity of figures:

- a) Check if the figures are similar, i.e. if they have proportional side lengths.
- b) If so, use the similarity properties of the figures, such as ratios of side lengths, angles of similarity, etc., to calculate the missing lengths or angles.

Analytical Geometry Problem Solving:

- a) Present the geometric figures in the coordinate system, assigning coordinates to the points.
- b) Use mathematical analysis tools such as distance formulas, equations of lines, tangents, etc. to calculate figure properties such as side lengths, angles, area, etc.

Solving problems with difficult geometric figures:

- a) Analyze the drawing, identify the data and values you are looking for.
- b) Use the properties and theorems of geometry such as Thales' theorem, the law of sines, the law of cosines, etc. to find answers to questions.

Solving tasks in spatial geometry:

- a) Recognize three-dimensional geometric figures such as cubes, pyramids, cones, etc.
- b) Use formulas and theorems about volumes, areas, diagonals, etc. to calculate missing values or relationships between the elements of a figure.

Work systematically and carefully:

- a) Draw exact drawings, lengths and angles of the labels.
- b) Use appropriate theorems and formulas that apply to the given figure or problem.
- c) Carefully analyze data, apply logic and precise reasoning when solving problems.

Remember that the most important thing is practice in learning geometry. The more problems you solve, the better you will master different techniques and approaches to geometric problems.

A set of complicated geometry problems:

1. Triangle similarity problem:

In triangle ABC, DE is parallel to BC. The length of the line AD is 6 cm and the length of the line DB is 4 cm. Calculate the ratio of the length of segment AE to the length of segment EC.

2. Equilateral triangle problem:

A circle is inscribed in an equilateral triangle ABC of side length 12 cm. Find the length of the arc AB that is simultaneously circumscribed about this circle.

3. Rectangular problem:

In a cuboid ABCDEFGH with side AB = 6 cm, the diagonal BG is 10 cm long. Calculate the angle between the CDH plane and the ADG plane.

4. Parallel lines problem:

There are three parallel lines: l, m, and n. The distance between the lines l and m is 5 cm, and the distance between the lines l and n is 3 cm. Calculate the distance between lines m and n.

5. A sphere inscribed in a tetrahedron problem:

A sphere is inscribed in the tetrahedron ABCD. Point E is the midpoint of edge AB. Calculate the ratio of the volume of a sphere to the volume of a tetrahedron.

6. Problem for a triangle internally touching a circle:

In triangle ABC, the inscribed circle touches sides AB, BC, and CA at points D, E, and F, respectively. AB is 10 cm long, BC is 12 cm, and CA is 14 cm. Calculate the length of the segment EF.

7. Regular triangle problem:

In a triangle ABC with side AB = 12 cm, points D, E, and F are the midpoints of the sides BC, CA, and AB, respectively. Calculate the ratio of the area of triangle DEF to the area of triangle ABC.

8. Cone and sphere problem:

A sphere is inscribed in a cone of height 8 cm and base radius 6 cm. Calculate the ratio of the volume of the cone to the volume of the sphere.

9. Regular quadrilateral problem:

In a quadrilateral ABCD with sides $AB = 6$ cm, $BC = 8$ cm, $CD = 10$ cm, and $DA = 12$ cm, the points E, F, G, and H are the midpoints of the sides AB, BC, CD, and DA, respectively. Calculate the ratio of the area of the quadrilateral EFGH to the area of the quadrilateral ABCD.

10. Pythagorean triangle problem:

In triangle ABC, angle A is right and side AB is 5 cm. The side BC is equal to the product of the length of the side AB and some integer k. If the area of triangle ABC is 15 cm^2 , find the value of k.

The answers to a set of complicated geometry problems:

1. Answer: The ratio of the length of segment AE to the length of segment EC is 2:3.
2. Answer: The length of the arc AB is 4π cm.
3. Answer: The angle between the CDH plane and the ADG plane is approximately 52.32 degrees.
4. Answer: The distance between the lines m and n is 4 cm.
5. Answer: The ratio of the volume of a sphere to the volume of a tetrahedron is 1:3.

6. Answer: The length of the segment EF is $2\sqrt{15}$ cm.
7. Answer: The ratio of the area of triangle DEF to the area of triangle ABC is 1:9.
8. Answer: The ratio of the volume of a cone to the volume of a sphere is $3:2\pi$.
9. Answer: The ratio of the area of the quadrilateral EFGH to the area of the quadrilateral ABCD is 1:4.
10. Answer: The value of k is 6.

Please note that the answers may vary depending on the accuracy of the calculations and the assumptions made. When solving complex geometric problems, it is always worth providing full calculations and justifications to prove the correctness of the answer.

